Magnetic flux in a mesoscopic SQUID controlled by nonclassical electromagnetic fields

Jerzy Dajka and Jerzy Łuczka
Institute of Physics, University of Silesia, 40-007 Katowice, Poland

(Received 1 September 2009; revised manuscript received 2 November 2009; published 30 November 2009)

We analyze SQUID coupled to a nonclassical electromagnetic field (NEM) and show that properties of the SQUID can be manipulated by a choice of a state of NEM. In particular, energy or fluctuations of magnetic flux threading the loop of the SQUID can be resolved into separate lines for each photon number state of one-mode NEM. The impact of two-mode NEM prepared in entangled Bell states is discussed. The findings suggest an experimental method of detection of photon states: the SQUID response is dependent on the number of photons in one-mode NEM and on the Bell states of a two-mode NEM.

DOI: 10.1103/PhysRevB.80.174529 PACS number(s): 85.25.−j, 74.50.+r

I. INTRODUCTION

The development of quantum information processing and quantum engineering results in growing interest in systems which are in a border of solid state and quantum optics.[1] Recently fabricated mesoscopic devices interacting with light are sensitive to their quantum properties.[2,3] One of them, which is of central interest of this paper, is resistively shunted AC SQUID.[4,5] In this paper, we focus our attention on magnetic flux and current flowing in the SQUID, which is coupled to an external nonclassical electromagnetic field.[6] By nonclassical we mean the electromagnetic field prepared in a given state defined by a statistical operator ρ. As an example one can take the states of recently produced light of both microwave and optical frequencies.[7] The problem of influence of classical electromagnetic fields on activation processes in Josephson junction has been already studied in Ref. 8. It has also been shown that the supercurrent flowing in the SQUID is highly responsive to properties of nonclassical radiation.[6] Moreover, the investigation of the impact of such radiation on persistent currents flowing in mesoscopic metallic and semiconducting rings has been carried out.[9,10]

Below we construct a model for kinetics of effective magnetic flux in SQUID and we find that properties of magnetic flux in a stationary state are related to quantum features of the nonclassical electromagnetic field (NEM). We shall consider both one-mode (number eigenstates and thermal states) and two-mode fields. In the later case we compare and contrast properties of the flux in presence of each of four maximally entangled Bell states[11] of the ground and first excited number eigenstates. Let us notice that within proposed model studying classical properties of magnetic flux in the SQUID, one can speculate about quantum character of the electromagnetic fields. SQUID rings being nowadays relatively accessible laboratory equipment can serve as devices measuring certain aspects of the nonclassical radiation.

The layout of the present work is as follows: in Sec. II, we present the model of the superconducting loop interrupted by a Josephson junction and subject to the external NEM. Next, in Sec. III, we elucidate impact of a one-mode NEM on the system and analyze stationary states of the magnetic flux. In Sec. IV, we work out the influence of two-mode NEM. In Sec. V, we elaborate on the vacuum and thermal effects. Sec. VI provides a summary and some conclusions.

II. MODEL

Let us consider a superconducting loop (ring, cylinder, and torus) interrupted by a Josephson junction. It is a basic element of the SQUID. The phase difference ψ of the Cooper pair wave function across the junction is related to the magnetic flux ϕ threading the loop via the relation

$$\psi = 2\pi(n - \phi/\phi_0), \tag{1}$$

where $2\pi n$ is the phase change per cycle around the loop and $\phi_0 = h/2e$ is the flux quantum. When the external magnetic field is applied, the actual flux is given by

$$\phi = \phi_e + LI, \tag{2}$$

where ϕ_e is the flux generated by an applied classical field, L is the selfinductance of the loop and I is the loop current. We model the Josephson element in terms of the resistively and capacitively shunted junction for which the current splits into three contributions, namely,

$$I = I_J + I_R + I_C, \tag{3}$$

where I_J is the Josephson supercurrent, I_R is a normal (Ohmic) current characterized by the normal state resistance R and I_C is a displacement current accompanied with the junction capacitance C.

Josephson junctions are influenced by electromagnetic fields in two ways. First, a time-dependent vector potential changes the spatial phase distribution in the Josephson junction and leads to the observation of the magnetic field dependence of the critical current in the Fraunhofer pattern. Second, an alternating bias current with a frequency of order of the Josephson frequency leads to many interesting nonlinear phenomena including the constant voltage steps in the current voltage characteristics, i.e., the Shapiro steps. The frequency range of observation extends to energies of the order of the superconducting gap.

When the electromagnetic field is treated as a classical one, dynamics of the system is described by a Langevin equation describing a Brownian particle in a sinusoidaly modulated parabolic potential. Now, let additionally the quantum (nonclassical) monochromatic electromagnetic field of frequency ω be applied. It induces the nonclassical flux ϕ_q of the form[12]

$$\phi_q = \phi_1[\exp(i\omega t)a^\dagger + \exp(-i\omega t)a], \quad (4)$$

where ϕ_1 is its amplitude, $a^\dagger$ and a are the creation and annihilation operators of the nonclassical electromagnetic field.

The full quantum model, quantizing both the electromagnetic field and the Josephson junction, has been studied in a series of papers.[6,12,13] If the frequency ω is much larger than the plasma frequency ω_p of the Josephson junction, dynamics of the junction is much slower than changes of the electromagnetic field. In such a case, the leading quantum corrections induced by the nonclassical flux can be incorporated in the Josephson supercurrent. The generalized method of elimination of fast variables results in the following expression for the Josephson supercurrent[6]

$$I_J \to -I(\phi,t)$$

$$I(\phi,t) = I_0 \,\mathrm{Im}\left\{\exp\left(2\pi i\frac{\phi}{\phi_0}\right)\mathrm{Tr}\left[\rho\Delta\left(2\pi\frac{\phi_1}{\phi_0}e^{i\omega t}\right)\right]\right\}, \quad (5)$$

where I_0 is the critical supercurrent, ρ is the density matrix of the nonclassical electromagnetic field and the operator $\Delta(A)=\exp(Aa^\dagger - A^* a)$ is the displacement operator.

Combining Eqs. (1)–(5) with the Josephson second relation $d\psi/dt=2eU/\hbar$, where U is the voltage drop across the junction, we get the Langevin-type equation

$$C\frac{d^2\phi}{dt^2} + \frac{1}{R}\frac{d\phi}{dt} = F(\phi,t) + \sqrt{\frac{2k_BT}{R}}\Gamma(t) \quad (6)$$

This stochastic equation has a simple mechanical interpretation: it describes the instantaneous "position" $\phi(t)$ of the "Brownian particle" evolving under the position–dependent force $F(\phi,t)$ in the presence of random fluctuations $\Gamma(t)$ resulting from surroundings. The generalized "force" acting on the "particle" at the position ϕ is given by the sum of two terms

$$F(\phi,t) = -\frac{\phi - \phi_e}{L} + I(\phi,t). \quad (7)$$

The first term originates from the self-inductive interaction of the magnetic flux whereas the second term is the supercurrent modified by the quantum flux. The ubiquitous thermal equilibrium noise $\Gamma(t)$ consists of Johnson noise associated with the resistance R. The parameter k_B denotes the Boltzmann constant and T is temperature of the system. The Johnson noise is modeled by δ-correlated Gaussian white noise of zero mean and unit intensity, i.e. $\langle\Gamma(t)\Gamma(u)\rangle = \delta(t-u)$.

In order to have a sensitive enough system, which feels the quantum nature of the flux ϕ_q, the junction should be mesoscopic. To consider this case, let us rescale Eq. (6) to the dimensionless form. The dimensionless flux is defined as $x=\phi/\phi_0$ and the dimensionless time is chosen in the form $s=t/\tau_0$, where the characteristic time $\tau_0=L/R$. Then Eq. (6) is converted to the dimensionless form

$$\epsilon\ddot{x} + \dot{x} = f(x,s) + \sqrt{2D}\Gamma(s), \quad (8)$$

where a dot denotes the differentiation with respect to the rescaled time s and 'inertial' effects are controlled by the dimensionless parameter $\epsilon=CR^2/L=\tau_1/\tau_0$ which is a ratio of two characteristic times $\tau_1=RC$ and $\tau_0=L/R$. The "force" $f(x)$ takes the form

$$f(x,s) = -(x-x_e) + i_0\,\mathrm{Im}[\exp(2\pi i x)\mathrm{Tr}(\rho\Delta(\xi e^{i\Omega s})\,], \quad (9)$$

where the external flux scales as $x_e=\phi_e/\phi_0$, the dimensionless critical current $i_0=LI_0/\phi_0$ and the effects of quantum flux are determined by the rescaled amplitude $\xi=2\pi\phi_1/\phi_0$ and the rescaled frequency $\Omega=\omega\tau_0$ of the nonclassical electromagnetic field. The rescaled noise intensity $D=k_BT/2\mathcal{E}$ is a ratio of two characteristic energies, i.e., thermal fluctuation energy $k_BT/2$ and the characteristic elementary magnetic flux energy $\mathcal{E}=\phi_0^2/2L$.

For very small (mesoscopic) junctions, the parameter $\epsilon \ll 1$ (i.e.,$\tau_1 \ll \tau_0$) and the capacitance term can be neglected. It is the case when Eq. (8) reduces to the "overdamped" form

$$\dot{x} = f(x,s) + \sqrt{2D}\Gamma(s). \quad (10)$$

Having formulated the model in terms of a stochastic dynamical system [Eq. (10)] we study it via the corresponding Fokker-Planck equation for the probability density $p(x,s)$ of the magnetic flux x at time s. It reads

$$\frac{\partial}{\partial s}p(x,s) = -\frac{\partial}{\partial x}f(x,s)p(x,s) + D\frac{\partial^2}{\partial x^2}p(x,s). \quad (11)$$

It is a common practice in studies of dynamical systems to introduce the "generalized potential"

$$V(x,s) \equiv -\partial f(x,s)/\partial x, \quad (12)$$

which is particularly useful for analyzing stationary properties of autonomous systems, i.e., such that $V(x,s)\equiv V(x)$. The solution of the Fokker-Planck equation carries all statistical informations concerning the process $x(t)$ determined by Eq. (10). We limit our consideration to analyze of the first two statistical moments. The first moment $\langle x\rangle$ describes the average value of the magnetic flux in the SQUID. The second moment $\langle x^2\rangle$ is directly related to the mean magnetic energy storied in the field produced by the SQUID. The quantity of central interest due to its information-theoretical applications is the celebrated Shannon entropy $S[p] = -\int p(x,s)\log[p(x,s)]dx$. It indicates the disorder introduced into the system by nonclassical electromagnetic fields.

The effect of the external NEM enters via the Weyl function

$$W(\xi,s) = \mathrm{Tr}[\rho\Delta(\xi e^{i\Omega s})] \quad (13)$$

modifying the force $f(x,s)$ and, as a result, the effective kinetics of the magnetic flux $x(s)$ in the superconducting loop. Therefore the state of the SQUID can be manipulated by the state ρ of the nonclassical electromagnetic field. There are two classes of states ρ. For the first class, the Weyl function does not depend on time, i.e., $W(\xi,s)=W(\xi)$. For the second class, the Weyl function does depend on time. The examples of the members of the first class are the number eigenstates, thermal states, coherent states with randomized phase. The second class includes ordinary coherent states, squeezed states, and squeezed vacua. For members of the first class, the generalized potential $V(x,s)$ does not depend on time, i.e., $V(x,s)=V(x)$. In consequence, in the long time limit, the system reaches a unique stationary state $p_{st}(x)$ independent on its initial state, i.e., $p_{st}(x)=\lim_{s\to\infty} p(x,s)$. In particular, the magnetic flux takes its stationary values, which in general depends on the state ρ of NEM. Below, as an example we consider the SQUID with the rescaled $i_0=\xi=1$, $x_e=0$ and temperature such that $D=0.1$ (such choice ensures that the effect of nonclassical radiation is minimally obscured by thermal effects). For $x_e=0$, properties of the current flowing in the ring are similar to the properties of the actual magnetic flux because in this case $\phi=LI$, cf. Eq. (2).

III. SINGLE-MODE FIELDS

In this section, we consider the stationary magnetic flux in the SQUID irradiated by monochromatic microwaves prepared in number eigenstates

$$\rho = |N\rangle\langle N|, \quad N=0,1,2,\ldots \tag{14}$$

Number eigenstates belong to the class of states having no well defined phase, the other representative of this class are e.g., coherent states with randomized phase.[6] Contrary to the states having a well defined phase such as coherent states, the number eigenstates do not introduce explicit time dependence to the generalized potential [Eq. (12)] because the Weyl function has the form[6,14]

$$W(\xi) = \exp\left(-\frac{1}{2}\xi^2\right)L_N(\xi^2), \tag{15}$$

where L_N are Laguerre polynomials of a degree N.[14] The plot of the potential Eq. (12) parameterized by the number of excitations N is given in Fig. 1. The structure of the generalized potential in the autonomous systems is reflected in the properties of the steady state, i.e., $p_{st}(x)\sim\exp[-V(x)]$. We discuss in the following the specific problem of bistability of the asymptotic state. The system in absence of external fields is bistable since its dynamics is governed by a double-well potential. This structure is preserved for NEM in its ground state $|N\rangle=|0\rangle$. Increasing the number of excitations in the NEM results in a single well, monostable system for, e.g., $N=1$. There is a transient regime around $N=12$ where the system becomes bistable again. This property vanishes for higher N. Let us notice that this property highly depends on the amplitude ξ of the nonclassical field. Increasing the amplitude results in an effective exponential lowering of the amplitude of nonlinear term in Eq. (9),

$$i_0 \to i_0 \exp\left(-\frac{1}{2}\xi^2\right)L_N(\xi^2) \tag{16}$$

and, in the formal limit $\xi\to\infty$ the potential becomes harmonic. This limit can be approximated via increasing the area of the SQUID loop, i.e., approaching the limit of a classical RL circuit. It is clear that this behavior is reflected in statistical properties of the magnetic flux. The external nonclassical control does not change the expectation value of the

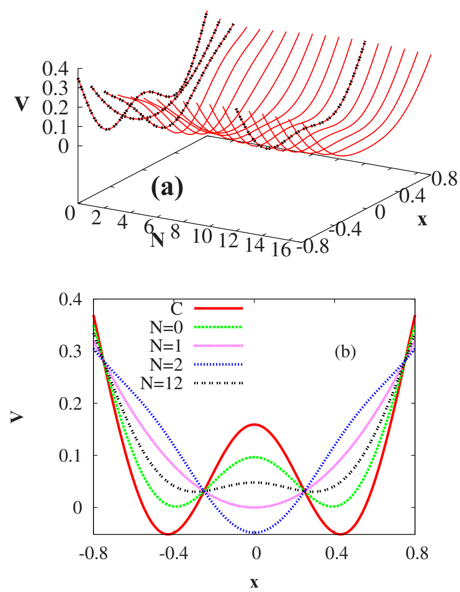

FIG. 1. (Color online) The potential $V(x)$ in the presence of number eigenstates. The curve labeled as C in panel (b) represents the system in an empty space ('classical vacuum'). The "highlighted" potentials in panel (a) correspond to $N=0,1,2,12$.

flux and the stationary averaged value $\langle x\rangle=0$. It is not the case for higher moments. In the panel (a) of Fig. 2, we plot the fluctuations of the magnetic flux produced by the SQUID in the presence of electromagnetic control. There are two qualitatively different regimes: in the first one, the external control results in lowering of fluctuations, whereas in the second one, the fluctuations increase due to the radiation. It

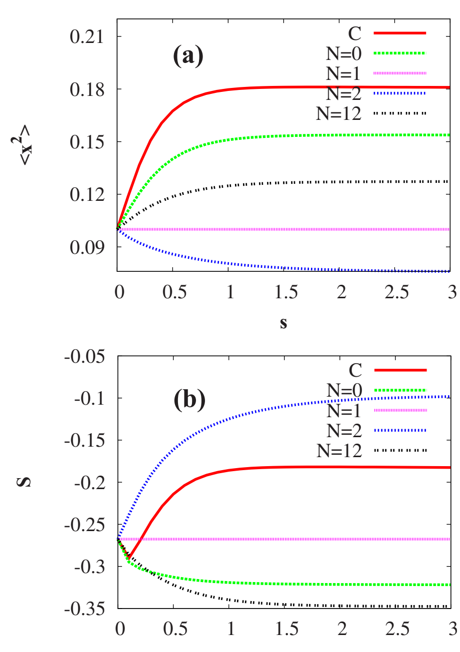

FIG. 2. (Color online) The second moment and the entropy of the magnetic flux in the SQUID irradiated by a nonclassical electromagnetic field in the number eigenstates. The initial state is assumed Gaussian, i.e., initially the distribution of the magnetic flux is such that $p(x,0)\sim N(0,0.1)$.

is directly related to the disorder introduced in the system as can be deduced from the panel (b) of Fig. 2 where the Shannon entropy is presented. The very specific character of kinetics for $N=1$ depends essentially on the initial state $p(x,0)$. The nonmonotonic character of the entropy for the "purely classical" case is directly related to the formation of dynamic bimodality[15] in the SQUID. The stationary averaged energy of the magnetic flux $E_\phi = \langle \phi^2 \rangle / 2L$ and in its dimensionless form is related to the second statistical moment, i.e., $E_\phi \propto = \langle x^2 \rangle$. From Fig. 2 it follows that E_ϕ contains information about the state of NEM. Indeed, E_ϕ can be resolved into separate lines for each photon number state of NEM. However, the dependence of $E_\phi(N)$ is not monotonic with respect to the number N of photons.

IV. TWO-MODE ENTANGLED STATES

Entangled states are of central importance for both applications and fundamentals of quantum theory. They can serve as a resource for most of the protocols known in quantum information processing[16] using entangled states of desired properties. The model discussed in the previous sections can easily been extended to the case of two-mode NEM of the same amplitude ξ by introducing another set of bosonic operators generating their quanta.[9,10,17] In this section we discuss sensitivity of the dissipative SQUID to such states. We consider here the set of four maximally entangled states of two-mode system consisting of two subsystems limited to their ground and first excited state:[11,16]

$$|B1\rangle = \frac{1}{\sqrt{2}}[|01\rangle + |10\rangle],$$

$$|B2\rangle = \frac{1}{\sqrt{2}}[|01\rangle - |10\rangle],$$

$$|B3\rangle = \frac{1}{\sqrt{2}}[|00\rangle + |11\rangle],$$

$$|B4\rangle = \frac{1}{\sqrt{2}}[|00\rangle - |11\rangle]. \tag{17}$$

The corresponding Weyl functions $W(\xi, s \equiv W(B)$ entering the "force" and the generalized potential $V(x,s)$ in Eq. (12) have been calculated e.g., in Refs. 9 and 10 and read

$$W(B1) = \exp(-\xi^2)\left[\frac{1}{2}(1-\xi^2) - \xi^2\cos(\Omega_- s)\right],$$

$$W(B2) = \exp(-\xi^2)\left[\frac{1}{2}(1-\xi^2) + \xi^2\cos(\Omega_- s)\right],$$

$$W(B3) = \exp(-\xi^2)\left[\frac{1}{2} + \frac{1}{2}(1-\xi^2) + \xi^2\cos(\Omega_+ s)\right],$$

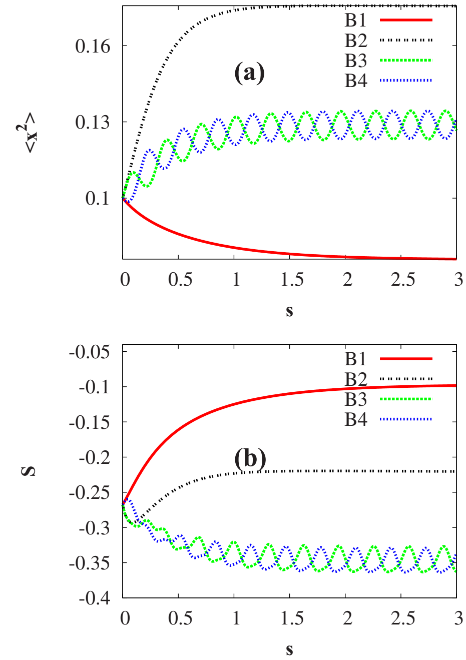

FIG. 3. (Color online) The second moment and the entropy of the magnetic flux in the SQUID irradiated by a nonclassical two-mode electromagnetic field in the entangled number eigenstates. $\omega_1 = \omega_2 = 10$. The initial state $p(x,0)$ is the same as in Fig. 2.

$$W(B4) = \exp(-\xi^2)\left[\frac{1}{2} + \frac{1}{2}(1-\xi^2) - \xi^2\cos(\Omega_+ s)\right], \tag{18}$$

with $\Omega_\pm = \omega_1 \pm \omega_2$ and ω_i, $i=1,2$ are the frequencies of i-th mode.

The external nonclassical radiation prepared in maximally entangled Bell states results in a nonautonomous correction to kinetics of the magnetic flux. Because the Weyl functions depend on time, the generalized potential $V(x,s)$ also depends on time. Therefore, in the long time limit, the averaged magnetic flux depends on time and is a periodic function of time. Only in a particular case when $\omega_1 = \omega_2$, two Weyl functions $W(B1)$ and $W(B2)$ do not depend on time. Therefore, entangled states $|B1\rangle$ and $|B2\rangle$ can be distinguishable by measurement of the stationary second moment of the magnetic flux, i.e., the stationary averaged flux energy. Unfortunately, the entangled states $|B3\rangle$ and $|B4\rangle$ cannot be separately resolved from statistics of the magnetic flux. We visualize it in Fig. 3, where time dependence of the second moment $\langle x \rangle$ and the entropy are depicted as a function of time. For long times, the influence of the states $|B1\rangle$ and $|B2\rangle$ are clearly different and it could allow to distinguish two entangled states of two-mode NEM.

V. THERMAL EFFECTS IN NEM

Effects of nonzero temperature and dissipation have been included only in the *classical* part of our system. For the SQUID, we use a Langevin-type equation with dissipative and noise terms according to the fluctuation-dissipation theo-

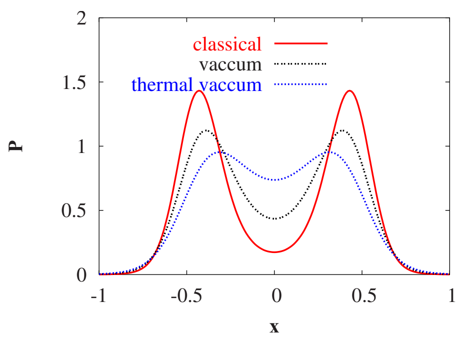

FIG. 4. (Color online) Stationary probability density of the magnetic flux in the SQUID in different vacuums: classical empty space, nonthermal $|0\rangle$, and thermal Eq. (19). For thermal vacuum it is set $\hbar\omega/k_BT=1$.

rem. The state of the quantum electromagnetic field remains unchanged and pure. Such a state carries no classical noise.[17] Clearly it is not the case in real experimental devices where one cannot separate the environment from a part of the total system. The form of an arbitrary state of radiation in the presence of the thermal light is known.[18] The formulas are rather complicated. The corresponding Weyl functions need a numerical treatment. Here, we limit our considerations to the simplest case of the so called thermal vacuum

$$\rho = (1 - e^{-\hbar\omega/k_BT})\sum_N e^{-N\hbar\omega/k_BT}|N\rangle\langle N| \tag{19}$$

and study the asymptotics of the *ground state* of the SQUID in the presence of nonclassical radiation. We compare it with classical one, i.e., with that in empty space. We incorporate into our discussion both charging effects due to small size of the SQUID and thermal noise present in the radiation which state becomes *thermal*. The corresponding Weyl function

$$W = \exp\left[-\frac{1}{2}\xi^2 \coth\left(\frac{1}{2}\hbar\omega/k_BT\right)\right]. \tag{20}$$

It is clear that the thermal effects added to the vacuum state result in further smearing of the probability density of magnetic flux as presented in Fig. 4. Other statistical characteristics of the magnetic flux in the SQUID calculated via the simplified phenomenological model proposed in this paper are affected in a qualitatively similar way. As a result any measurement of the magnetic flux may be obscured when the interesting phenomena (such as e.g., oscillations of the second moment of the magnetic flux related to the Bell states of two-mode NEM discussed in Sec. IV) occur below the "sensitivity threshold" of measuring apparatus. Hence, it is of great importance to reduce thermal effects in NEM as much as possible.

To be more consistent in the description of the full system, we should use two approaches and next test them. The first, using the Langevin equation for SQUID and adding a coupling of the quantum field to a thermal bath. The second could be formulated as a SQUID+NEM quantum system in which both parts are coupled to a common quantum thermal bath. Fully consistent description of the thermal effects in both the SQUID and NEM is essentially beyond the proposed semiphenomenological model of magnetic flux kinetics and would require a treatment based e.g., on the Caldeira-Leggett type model of the SQUID+NEM+thermal bath.[5] In the case of weak coupling to a heat bath, one could employ a Born-Markov approximation and exploite a standard master equation of the Lindblad-Kossakowski form.[19]

VI. SUMMARY

The magnetic flux in the superconducting loop interrupted by a Josephson junction can be controlled by external fields. The kinetic properties of the flux are to some extent governed by such a driving and, as a result, can become useful for studying driving characteristics by indirect methods. It is of great importance in the case when a driving is a nonclassical electromagnetic field. In such a case, properties of the magnetic flux can reflect quantum nature of radiation. The properties are primarily encoded in the state of the NEM. Unfortunately, their measurement is often difficult and highly sophisticated.[16] The celebrated quantum tomography is here a well known example.[16] The considered "SQUID & NEM system" belongs to a class of hybrid quantum-classical systems.[20,21] Contrary to Ref. 21, in this paper, we are interested in kinetics of its classical part. Our proposal may be useful for constructing devices measuring certain characteristics of nonclassical fields allowing to extract *a part* of information encoded in the state of nonclassical field. Measurement protocols based on detection of energy or fluctuations of the magnetic flux in SQUID could allow to diagnose quantum field states with different photon statistics and discrete nature of NEM.

ACKNOWLEDGMENT

The work supported by the Polish Ministry of Science and Higher Education under Grant No. N 202 131 32/3786.

[1] Y. Yamamoto and A. Imamoglu, *Mesoscopic Quantum Optics* (Wiley, New York, 1999).
[2] T. Brandes, Phys. Rep. **408**, 315 (2005).
[3] D. I. Schuster, A. A. Houck, J. A. Schreier, A. Wallreff, J. M. Gambetta, A. Blais, L. Frunzio, J. Mayer, B. Johnson, M. H. Devoret, S. M. Girvin, and R. J. Schoelkopf, Nature (London) **445**, 515 (2007).
[4] A. Barone and G. Paterno, *Physics and Applications of the Josephson Effect* (Wiley, New York, 1982).
[5] U. Weiss, *Quantum Dissipative Systems* (World Scientific, Singapore, 1999).
[6] A. Vourdas, Phys. Rev. B **49**, 12040 (1994).
[7] R. Loudon and P. L. Knight, J. Mod. Opt. **34**, 709 (1987); R. Loudon, *The Quantum Theory of Light* (Oxford University

Press, New York, 2000); D. F. Walls and G. Milburn, *Quantum Optics* (Springer-Verlag, Berlin, 1994); B. Yurke, P. G. Kaminsky, R. E. Miller, E. A. Whittaker, A. D. Smith, A. H. Silver, and R. W. Simon, Phys. Rev. Lett. **60**, 764 (1988); B. Yurke, L. R. Corruccini, P. G. Kaminsky, L. W. Rupp, A. D. Smith, A. H. Silver, R. W. Simon, and E. A. Whittaker, Phys. Rev. A **39**, 2519 (1989).

[8] M. H. Devoret, D. Esteve, J. M. Martinis, A. Cleland, and J. Clarke, Phys. Rev. B **36**, 58 (1987).

[9] J. Dajka, M. Szopa, A. Vourdas, and E. Zipper, Phys. Rev. B **69**, 045305 (2004); J. Dajka, A. Vourdas, S. Zhang, and E. Zipper, J. Phys.: Condens. Matter **18**, 1367 (2006).

[10] J. Dajka, M. Szopa, M. A. Vourdas, and E. Zipper, Phys. Status Solidi B **242**, 296 (2005).

[11] F. Mintert, A. R. R. Carvalho, M. Kus, and A. Buchleitner, Phys. Rep. **415**, 207 (2005).

[12] A. Vourdas and T. P. Spiller, Z. Phys. B **102**, 43 (1997).

[13] Zou Jian, Shao Bin, and Xing Xiu-San, Phys. Lett. A **231**, 123 (1997); Kuang Le-man, Wang Yi-wen, and Ge Mo-lin, Chin. Phys. Lett. **13**, 359 (1996).

[14] A. Perelomov, *Generalized Coherent States and Applications* (Springer-Verlag, Berlin, 1986).

[15] M. Mierzejewski, J. Dajka, J. Luczka, P. Talkner, and P. Hanggi, Phys. Rev. E **74**, 041102 (2006).

[16] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, England, 2000).

[17] A. Vourdas, Phys. Rev. A **64**, 053814 (2001).

[18] A. Vourdas, Phys. Rev. A **39**, 206 (1989).

[19] R. Alicki and K. Lendi, *Quantum Dynamical Semigroups and Applications*, Lecture Notes in Physics Vol. 286 (Springer, Berlin, 1987).

[20] A. Peres and D. R. Terno, Phys. Rev. A **63**, 022101 (2001).

[21] R. Kapral and G. Ciccotti, J. Chem. Phys. **110**, 8919 (1999).